NON-GAUSSIAN BLE-BASED INDOOR LOCALIZATION VIA GAUSSIAN SUM FILTERING COUPLED WITH WASSERSTEIN DISTANCE

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ABSTRACT

With recent breakthroughs in signal processing, communication and networking systems, we are more and more surrounded by smart connected devices empowered by the Internet of Thing (IoT). Bluetooth Low Energy (BLE) is considered as the main-stream technology to perform identification and localization/tracking in IoT applications. Indoor localization applications within smart cities, typically, start by observing messages transmitted by BLE beacons and then utilization of Received Signal Strength Indicator (RSSI) to provide location estimates. RSSI signals are, however, prone to significant fluctuations. The main challenge is that multipath fading and drastic fluctuations in the indoor environment result in complex non-Gaussian RSSI measurements, necessitating the need to smooth RSSIs for development of BLE-based localization applications. In contrary to existing solutions, where RSSIs are assumed to have normal statistical properties, in this paper, a Gaussian Sum Filter (GSF) approach is designed to more realistically model the non-Gaussian nature of RSSIs. To maintain acceptable computational load, the number of components in the GSF is collapsed into a single Gaussian term with a novel Wasserstein Distance (WD)-Based Gaussian Mixture Reduction (GMR) algorithm. The simulation results based on real collected RSSI signals confirm the success of the proposed WD-based GSF framework compared to its conventional counterparts.

***Index Terms*— Indoor Localization, Bhattacharyya Distance, Received Signal Strength Indicator (RSSI), Gaussian sum filter, Wasserstein Distance.**

1. INTRODUCTION

Internet-of-things (IoT)-based indoor positioning [1] for providing Location-based Services (LBSs), where user's exact location is obtained satisfying high accuracy requirements, has become of primary importance in recent years. Several LBSs such as navigation assistance in hospitals, localization in smart buildings, and tracking solutions in airports have been developed based on indoor positioning [2–4]. With recent developments in IoT technology, mobile devices like smartphones and tablets have been used in indoor localization contests. Global Positioning System (GPS) is best for outdoor localization. But due to decreasing the GPS signal strength in indoor environments, it does not work well in indoor localization systems. Due to the lower cost, lower energy consumption and the size of Bluetooth devices, Bluetooth Low energy (BLE)-based localization has attracted considerable attention as an effective alternative for indoor localization [5]. Received Signal Strength Indicator (RSSI)-based solutions, however, are prone to multipath fading and fluctuations in the indoor environment, which causes low accuracy. Consequently, solutions should be designed to purify the RSSI measurements. Fingerprinting (learning-based) [6,7] is a common method using for indoor localization, where use fingerprints to estimate target's position by matching online RSSI measurements to the closet offline location fingerprint [5, 8, 9]. K-Nearest Neighbors (K-NN) is one popular matching algorithms.

Literature Review: State estimation problem has been widely explored in target tracking [10] since Kalman filter (KF) appeared in 1960 [11]. Kalman filter is an estimator for linear state-space models with Gaussian additive noises, however, in the systems with non-Gaussian behaviour, KF cannot provide reliable results [12]. Non-Gaussian state estimation is a common phenomenon in most of real-valued systems. Gaussian mixture (GM) models are used for modeling non-Gaussian densities. For such problems, Gaussian Sum Filter (GSF) [12] are rich solutions to approximate the non-Gaussian noise with GMs model. While KF-based algorithms approximate the state probability density function (PDF) with a single Gaussian, GSF approximates the PDF with a weighted sum of Gaussian distributions. But, the number of GM components tends to increase exponentially during the time resulting in introducing of significant computational overhead. To tackle this challenge, at each step of GSF algorithm, GMs is approximates with lower number of Gaussian components by using the Gaussian mixture reduction (GMR) algorithms [13]. Generally speaking, the GMR algorithms are classified [15] into two main categories: (i) *Bottom-up approaches*, where the desire GM starts with a single Gaussian function and iteratively add more components to reach to the desired GMs model, and; (ii) *Top-down approaches*, where the number of Gaussian mixtures are decreased gradually either by removing an unimportant component or by using the clustering methods that merges the similar components. This reduction is done locally or globally based on different information-probabilistic similarity measures such as the Kullback–Leibler divergence (KLD) [14] or the Bhattacharyya distance (BD). Then the similar components are replaced by a single Gaussian component. However, both KLD and BD optimize the information gain achieved by GMR, in many applications like machine learning and computer vision, the Geometric shape of the GM is important. Wasserstein distance (WD) computes the geometric shape difference between two PDFs by minimizing the cost of transferring one PDF to another [10].

Contributions: The paper proposes a novel GMM-based proba-

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bilistic framework for assigning real-time observed RSSI vectors to different fingerprinting zones using Bhattacharyya distance [15]. More specifically, after measuring the RSSI values, the values at each zone is modelled with multivariate GM distribution. In the online phase, the proposed novel GSF algorithm is applied on the real-time RSSI values via incorporation of a Wasserstein distance (WD)-based clustering GMR for smoothing the RSSI values and simulating the non-Gaussian RSSI measurement noise as a GM models distribution. These distributions outputs are then compared in distribution with GMs, learned for each zone, via utilization of the BD [15]. Two scenarios are used for user's location estimation: (i) The BD distance between the output of the GSF and the trained GM associated with all the zones are calculated and the one with the minimum distance is identified as the target's zone, and; (ii) A weighted K-Nearest Neighbors (K-NN) algorithm is applied for location estimation, where the weights are computed based on the BD between the output of the GSF and the GMs distributions of K nearest zones to the identified one.

2. GMM-BASED INDOOR LOCALIZATION VIA BLE SENSORS

Throughout the paper, the following notations are used: Non-bold letter X denotes a scalar variable, lowercase bold letter $\boldsymbol{x}$ represents a vector, and capital bold letter $\boldsymbol{X}$ denotes a matrix. Transpose of a matrix $\boldsymbol{X}$ is denoted $\boldsymbol{X}^T$. Generally speaking, for indoor localization via fingerprinting, first the indoor venue is divided into the small zones.

During the offline phase, RSSI values are measured from the active BLE beacons across the surveillance venue including all the zones. In the online phase, upon collection of the RSSI vector at each measurement epoch, a similarity measure is used to compare the observed RSSI value with the ones associated with the fingerprinting zones learned during the offline phase, and select one as the predicted zone of the user. In this paper, the probability distribution of RSSI values are used during the online and offline phase to find this similarity. In other words, the RSSI values measured at each zone during the offline phase is modeled by a multivariate GMs and the RSSI vector received in the online phase is smoothed and modelled as a multivariate GMs by using the GSF based on the WD reduction methodology.

We consider tracking a user in an environment monitored with N_b BLE-enabled sensors. For fingerprinting, the venue is divided into N_{FP} grid of squares, where zone l is denoted by $\mathcal{Z}_l$ for ($1 \leq l \leq N_{\text{FP}}$). The center of each zone is referred to as a "training point".

The proposed GM-based localization framework consists of an offline phase and an online phase. The former consists of the following two main steps:

Step 1. Collecting N_{Train} number of raw RSSI vectors from N_b number of active beacons across Zone l, for ($1 \leq l \leq N_{\text{FP}}$), and constructing the following zone-specific training RSSI matrix

$$\boldsymbol{R}_l = \begin{bmatrix} RSSI^1_{(1,l)}, & \ldots, & RSSI^1_{(N_{\text{Train}},l)} \\ \vdots & \vdots & \vdots \\ RSSI^{N_b}_{(1,l)}, & \ldots, & RSSI^{N_b}_{(N_{\text{Train}},l)} \end{bmatrix}, \quad (1)$$

associated with Zone l, for ($1 \leq l \leq N_{\text{FP}}$).

Step 2. Modelling the N_{Train} number of RSSI vectors in matrix $\boldsymbol{R}_l$ with GM components, denoted by $P(\mathcal{Z}_l)$, consisting of n_c Gaussian components parameterized by $\theta_l^{(i)} = \{\phi_l^{(i)}, \boldsymbol{\mu}_l^{(i)}, \boldsymbol{\sigma}_l^{(i)}\}$, i.e.,

$$P(\mathcal{Z}_l) = \sum_{i=1}^{n_c} \phi^{(i)} \mathcal{N}(\boldsymbol{\mu}_l^{(i)}, \boldsymbol{\sigma}_l^{(i)}), \quad (2)$$

where $\phi_l^{(i)}$ is the i^{th} component's weight, $\boldsymbol{\mu}_l^{(i)}$ is the mean vector with its associated covariance matrix denoted by $\boldsymbol{\sigma}_l^{(i)}$.

It is worth mentioning that different from existing fingerprinting techniques, in the offline phase of the proposed framework, we model the RSSI values associated with the l^{th}-zone, for ($1 \leq l \leq N_{\text{FP}}$), as a multivariate GMM ($P(\mathcal{Z}_l)$) defined by Eq. (2). After completion of the offline phase, these learned statistics are used for real-time indoor localization as is described in the following sub-section.

2.1. The Online Phase

In the real-time implementation of the proposed algorithm at measurement epoch $k > 1$, first RSSI vector $\boldsymbol{z}_k = \left[Z_k^{(1)} \ldots Z_k^{(N_b)}\right]^T$ is constructed from measurements obtained from the N_b active beacons. Then, the GSF algorithm is applied on the observation vector $\boldsymbol{z}_k$ for the following two reasons:

(i) *Pre-processing*, which consists of RSSI smoothing to compensate the fading effects on the measured RSSI values, and;

(ii) *Construction of the Real-time GM Model*, where the observed and smoothed RSSI vector corresponding to the unknown location at iteration k is modeled with a GM.

The output of the GSF would then be utilized for pattern matching to find the user's zone via identifying the zone, which has the most similar distribution to that of the online measured RSSI vector. Finally, the following statistical assumptions are incorporated to develop the GSF [12]:

- It is assumed that initial distribution $P(\boldsymbol{x_0}|\boldsymbol{z_0})$ is equal to the prior density $P(\boldsymbol{x_0})$.
- It is assumed that the process noise $\boldsymbol{w}_k$ is an additive white Gaussian noise with known covariance matrices $\boldsymbol{Q} > 0$.
- The measurement noise is a non-Gaussian additive approximated by a mixture of m_c mixing components as

$$P(\boldsymbol{v}_k) = \sum_{i=1}^{m_c} \mu^{(i)} \mathcal{N}(\boldsymbol{v}^{(i)}, \boldsymbol{R}^{(i)}) \quad (3)$$

- It is assumed that measurement noise $\boldsymbol{v}_k$ is stable over all steps.

To formulate the WD-GSF algorithm, we define the state vector $\boldsymbol{x}_k = [\boldsymbol{y}_k, \Delta\boldsymbol{y}_k]^T$, which consists of RSSI value $\boldsymbol{y}_k$ and its associated rate of change denoted by $\Delta\boldsymbol{y}_k$. Term $\Delta\boldsymbol{y}_k$ is dependent on the environment and shows how the RSSI values are fluctuating. The higher the noise in the environment, the higher will be the fluctuation. We consider the following dynamic model to represent the evolution of the constructed state vector ($\boldsymbol{x}_k = [\boldsymbol{y}_k, \Delta\boldsymbol{y}_k]^T$) over time

$$\begin{bmatrix} \boldsymbol{y}_k \\ \Delta\boldsymbol{y}_k \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & \Delta t \\ 0 & 1 \end{bmatrix}}_{\boldsymbol{F}_k} \begin{bmatrix} \boldsymbol{y}_{k-1} \\ \Delta\boldsymbol{y}_{k-1} \end{bmatrix} + \begin{bmatrix} \boldsymbol{w}_k^{y} \\ \boldsymbol{w}_k^{\Delta\boldsymbol{y}} \end{bmatrix}, \quad (4)$$

and $\boldsymbol{w}_k = \mathcal{N}(0, \boldsymbol{Q})$, with its covariance matrix $\boldsymbol{Q}$ being learned during the offline phase, represents the forcing term of the model, which is a design parameter. For WD-GSF implementation, we consider the following observation model

$$\boldsymbol{z}_k = \underbrace{[1\ \ 0]}_{\boldsymbol{H}_k} \begin{bmatrix} \boldsymbol{y}_k \\ \Delta \boldsymbol{y}_k \end{bmatrix} + \boldsymbol{v}_k, \tag{5}$$

which $\boldsymbol{v}_k$ has the probability distribution in Eq. (3). The proposed WD-GSF approximates the likelihood function $p(\boldsymbol{z}_k|\boldsymbol{x}_k)$ with a GM as

$$P(\boldsymbol{z}_k|\boldsymbol{x}_k) \approx \sum_{i=1}^{m_c} w_k^{(i)} \mathcal{N}(\hat{\boldsymbol{z}}_{k|k-1}^{(i)}, \boldsymbol{S}_k^{(i)}), \tag{6}$$

where

$$\hat{\boldsymbol{z}}_{k|k-1}^{(i)} = \boldsymbol{H}_k \hat{\boldsymbol{x}}_{k|k-1} + \boldsymbol{v}^{(i)}, \tag{7}$$

$$\boldsymbol{S}_k^{(i)} = \boldsymbol{H}_k \boldsymbol{P}_{k|k-1} \boldsymbol{H}_k^T + \boldsymbol{R}_k^{(i)}, \tag{8}$$

and

$$w_k^{(i)} = \frac{\exp\{||\hat{\boldsymbol{z}}_{k|k-1}^{(i)} - \boldsymbol{z}_k||^2_{[\boldsymbol{S}_k^{(i)}]^{-1}}\}}{\pi^{N_x/2} |\boldsymbol{S}_k^{(i)}|^{1/2} \times \boldsymbol{\zeta}_k} \mu^{(i)}, \tag{9}$$

where

$$\boldsymbol{\zeta}_k = \sum_{i=1}^{m_c} \frac{\exp\{||\hat{\boldsymbol{z}}_{k|k-1}^{(i)} - \boldsymbol{z}_k||^2_{[\boldsymbol{S}_k^{(i)}]^{-1}}\}}{\pi^{N_x/2} |\boldsymbol{S}_k^{(i)}|^{1/2}} \mu^{(i)}. \tag{10}$$

Based on the Bayesian theory, Eq. (6) results in a GM model of the posterior distribution $p(\boldsymbol{x}_k|\boldsymbol{z}_k)$ as

$$P(\boldsymbol{x}_k|\boldsymbol{z}_k) \approx \sum_{i=1}^{m_c} w_k^{(i)} p(\boldsymbol{x}_k|\boldsymbol{z}_{k-1}) \mathcal{N}(\hat{\boldsymbol{z}}_{k|k-1}^{(i)}, \boldsymbol{S}_k^{(i)}). \tag{11}$$

Then, GMR reduction algorithm based on WD (Algorithm 1) is applied on Eq. (11) to collapse the resulting GM into one single Gaussian distribution $\mathcal{N}(\hat{\boldsymbol{x}}_{k|k}, \boldsymbol{P}_{k|k})$. In other words, WD-GSF is applied based on the following steps

Step 1. Prediction Step: Assuming that the posteriori distribution $P(\boldsymbol{x}_{k-1}|\boldsymbol{z}_{k-1})$ is collapsed into one single Gaussian denoted by $\mathcal{N}(\hat{\boldsymbol{x}}_{k-1|k-1}, \boldsymbol{P}_{k-1|k-1})$ via WD, the prediction step of the WD-GSF for computing $\hat{\boldsymbol{x}}_{k|k-1}$ is given by

$$\boldsymbol{P}_{k|k-1} = \boldsymbol{F}_k \boldsymbol{P}_{k-1|k-1} \boldsymbol{F}_k^H + \boldsymbol{Q}_k \tag{12}$$

$$\hat{\boldsymbol{x}}_{k|k-1} = \boldsymbol{F}_k \hat{\boldsymbol{x}}_{k-1|k-1} \tag{13}$$

Step 2. Filtering and Collapsing: In this step, WD-GSF simultaneously filters the m_c components identified in Eq. (6) and update the overall point estimation $\hat{\boldsymbol{x}}_{k|k}$ and its error covariance $\boldsymbol{P}_{k|k}$ by collapsing the m_c filtered components into one single Gaussian via Wasserstein-Based Clustering GMR algorithm.

In other words, $P(\boldsymbol{z}_k|\boldsymbol{x}_k)$ is a GM with m_c components and $P^{(i)}(\boldsymbol{z}_k|\boldsymbol{x}_k) = \mathcal{N}(\hat{\boldsymbol{z}}_{k|k-1}^{(i)}, \boldsymbol{S}_k^{(i)})$, for $(1 \leq i \leq m_c)$, represents the i^{th} Gaussian component of this GM model. The goal of WD-GSF is to approximate $P(\boldsymbol{z}_k|\boldsymbol{x}_k)$ with single Gaussian distribution $Q(\boldsymbol{z}_k|\boldsymbol{x}_k) = \mathcal{N}(\hat{\boldsymbol{z}}_{k|k-1}, \boldsymbol{S}_k)$. In contrary to the common approach of using the KLD for GMR purposes, we consider WD-based as the statistical measure. The WD is a symmetric and metric probabilistic similarity measure, which minimizes the cost of transferring a PDF to another one by considering the difference between the shapes of the underlying PDFs. The WD between the aforementioned two Gaussian components is measured as follows [10]

Algorithm 1 The Proposed WD-GSF

Input: $\boldsymbol{H}(\boldsymbol{x}), \boldsymbol{F}(\boldsymbol{x}), P(\boldsymbol{v}_k), \boldsymbol{z}_k, \boldsymbol{Q}_k, \hat{\boldsymbol{x}}_{k-1|k-1}, \boldsymbol{P}_{k-1|k-1}, \boldsymbol{R}_l (1 \leq l \leq N_{\text{FP}})$.
Output: $\hat{\boldsymbol{x}}_{k|k}, \boldsymbol{P}_{k|k}, \hat{\mathcal{Z}}_k$.
Prediction Step:
  $\boldsymbol{P}_{k|k-1} = \boldsymbol{F}_k \boldsymbol{P}_{k-1|k-1} \boldsymbol{F}_k^H + \boldsymbol{Q}_k$
  $\hat{\boldsymbol{x}}_{k|k-1} = \boldsymbol{F}_k \hat{\boldsymbol{x}}_{k-1|k-1}$
for $i = 1 : m_c$ **do**
  Compute $\hat{\boldsymbol{z}}_{k|k-1}^{(i)}, \boldsymbol{S}_k^{(i)}$ via Eq. (7) and Eq. (8) respectively.
  Compute $w_k^{(i)}$ via Eq. (9) and Eq. (10).
end for
$P(\mathcal{Z}_k) = P(\boldsymbol{z}_k|\boldsymbol{x}_k) \approx \sum_{i=1}^{m_c} w_k^{(i)} \mathcal{N}(\hat{\boldsymbol{z}}_{k|k-1}^{(i)}, \boldsymbol{S}_k^{(i)})$.
Collapse $P(\boldsymbol{z}_k|\boldsymbol{x}_k)$ into $Q(\boldsymbol{z}_k|\boldsymbol{x}_k) = \mathcal{N}(\hat{\boldsymbol{z}}_{k|k-1}, \boldsymbol{S}_k)$ with WD-based GMR.
Update Step:
  $\boldsymbol{K}_k = \boldsymbol{P}_{k|k-1} \boldsymbol{H}_k \boldsymbol{S}_k^{-1}$
  $\boldsymbol{P}_{k|k} = [\boldsymbol{I} - \boldsymbol{K}_k \boldsymbol{H}_k] \boldsymbol{P}_{k|k-1}$
  $\hat{\boldsymbol{x}}_{k|k} = \hat{\boldsymbol{x}}_{k|k-1} + \boldsymbol{K}_k (\hat{\boldsymbol{z}}_k - \hat{\boldsymbol{z}}_{k|k-1})$
Pattern Matching:
  Construct $P(\mathcal{Z}_l) = \sum_{i=1}^{n_c} \phi^{(i)} \mathcal{N}(\boldsymbol{\mu}_l^{(i)}, \boldsymbol{\sigma}_l^{(i)})$ for $(1 \leq l \leq N_{\text{FP}})$.
  First Scenario (S1):
    $\hat{\mathcal{Z}}_k = \arg\min_l \{d_b(\mathcal{Z}_k, \mathcal{Z}_l)\}$.
  Second Scenario (S2):
    $\hat{\mathcal{Z}}_k$ is estimated with weighted K-NN algorithm.

$$D_i = D_W^2(P^{(i)}(\boldsymbol{z}_k|\boldsymbol{x}_k), Q(\boldsymbol{z}_k|\boldsymbol{x}_k)) = \tag{14}$$
$$\text{tr}\{\boldsymbol{S}_k^{(i)} + \boldsymbol{S}_k - 2((\boldsymbol{S}_k^{(i)})^{1/2} \boldsymbol{S}_k (\boldsymbol{S}_k^{(i)})^{1/2})^{1/2}\} + ||\hat{\boldsymbol{z}}_{k|k-1}^{(i)} + \hat{\boldsymbol{z}}_{k|k-1}||_2^2.$$

To perform GMR on $P(\boldsymbol{z}_k|\boldsymbol{x}_k)$ with m_c components and reducing it to $Q(\boldsymbol{z}_k|\boldsymbol{x}_k)$ with $1 < m_c$ components WD (Eq. (14)) is used by first constructing an initial candidate for $Q(\boldsymbol{z}_k|\boldsymbol{x}_k)$. Then the components of $P(\boldsymbol{z}_k|\boldsymbol{x}_k)$ are associated with the closet components of $Q(\boldsymbol{z}_k|\boldsymbol{x}_k)$ based on the WD between the two components. The group of $P(\boldsymbol{z}_k|\boldsymbol{x}_k)$ components that are assigned to the same component of $Q(\boldsymbol{z}_k|\boldsymbol{x}_k)$ is called a cluster. Finally, the associated component of $Q(\boldsymbol{z}_k|\boldsymbol{x}_k)$ is replaced with the center of its cluster and its weight becomes equal to sum of the cluster members' weights.

After completion of the WD-based GMR component of the WD-GSF, the state vector is constructed as follows

$$\boldsymbol{K}_k = \boldsymbol{P}_{k|k-1} \boldsymbol{H}_k \boldsymbol{S}_k^{-1} \tag{15}$$

$$\boldsymbol{P}_{k|k} = [\boldsymbol{I} - \boldsymbol{K}_k \boldsymbol{H}_k] \boldsymbol{P}_{k|k-1} \tag{16}$$

$$\hat{\boldsymbol{x}}_{k|k} = \hat{\boldsymbol{x}}_{k|k-1} + \boldsymbol{K}_k (\hat{\boldsymbol{z}}_k - \hat{\boldsymbol{z}}_{k|k-1}) \tag{17}$$

After smoothing the RSSI values during the online phase ($\hat{\boldsymbol{x}}_{k|k}$), for pattern matching, against the excising solution, now we propose to measure (via statistical distant measures) the distance be-

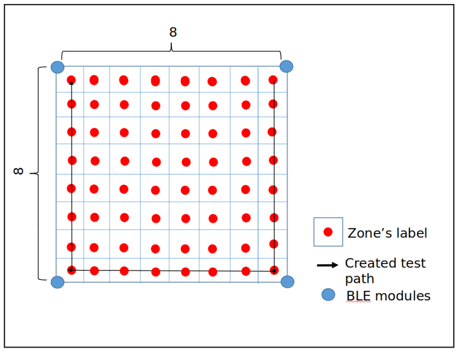
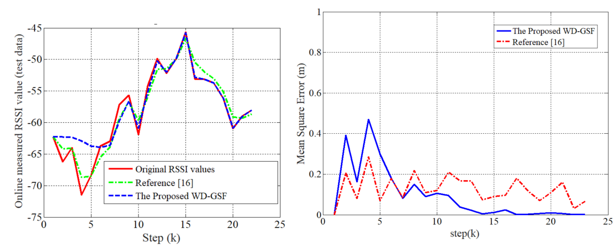


Figure 1: Data measurement setup.



(a) (b)

Figure 2: (a) RSSI smoothing resulting from conventional KF and GSF. (b) MSE comparison between KF-based algorithm and WD-GSF algorithm.

tween two probability distributions, i.e.,

$$P(\mathcal{Z}_l) = \sum_{i=1}^{n_c} \phi^{(i)} \mathcal{N}(\boldsymbol{\mu}_l^{(i)}, \boldsymbol{\sigma}_l^{(i)}),$$

$$\text{and} \quad P(\mathcal{Z}_k) = P(\boldsymbol{z}_k|\boldsymbol{x}_k) = \sum_{i=1}^{m_c} w_k^{(i)} \mathcal{N}(\hat{\boldsymbol{z}}_{k|k-1}^{(i)}, \boldsymbol{S}_k^{(i)}).$$

We use BD as follows

$$d_b(P(\mathcal{Z}_k), P(\mathcal{Z}_l)) = -\ln\Big(\int_{\mathbb{R}^{N_b}} \sqrt{P(\mathcal{Z}_k)P(\mathcal{Z}_l)}\, d\boldsymbol{z}\Big). \qquad (18)$$

Based on the BD specified in Eq. (18), the WD-GSF can be implemented in terms of either of the following two variants:

S1. The first estimation scenario is applied by computing the BD between the measurement vector $\boldsymbol{z}_k$ and all the N_{FP} GM distributions and associating the one with smallest distance, i.e. $\hat{\mathcal{Z}}_k = \arg\min_l \{d_b(\mathcal{Z}_k, \mathcal{Z}_l)\}$.

S2. In the second estimation scenario, first K nearest neighbors to the observation $\boldsymbol{z}_k$, between all fingerprinting zones are detected based on their corresponding BD. Then, weighted average of the locations of the K selected neighbours, with weights computed as follows $\frac{1}{d(\boldsymbol{Z}_k, \mathcal{Z}_l)}$, is considered as the user's location

This complete description of the proposed WD-GSF framework, which is summarized in Algorithm 1.

3. EXPERIMENTAL RESULTS

The two different variants of the proposed WD-GSF are applied and evaluated based on a real dataset consisting of RSSI measurements collected in a $(4.0m \times 4.0m)$ room divided into 64 square

Table 1: Accuracy comparison of first location estimation algorithms for various number of GMM components.

# of Components (n_c)	**GM**	**2-GMM**	**4-GMM**	**8-GMM**
GM	0.40	0.45	0.25	0.25
2-GMM	0.36	**0.62**	0.45	0.30

Table 2: Accuracy comparison of second location estimation algorithms with $K = 5$ for various number of GMM components.

# of Components (n_c)	**GM**	**2-GMM**	**4-GMM**	**8-GMM**
GM	0.88	0.90	0.88	0.74
2-GMM	0.74	**0.95**	0.80	0.62

zones with dimension $(0.5m \times 0.5m)$ as shown in Fig. 1. For fingerprinting, $1,000$ RSSI vectors were measured at each zone while 4 BLE sensors were located in the corners of the venue with 4m distance from each others. The constructed dataset together with its description can be accessed through its web-page: http://i-sip.encs.concordia.ca/datasets.html.

Similar to our previous work done in Reference [16], for evaluation of the proposed GMM-based approaches, a trajectory is created (as shown in Fig. 1) based on 22 selected test points covering the area for which test RSSI values are computed. We have $1,000$ offline measured RSSI vectors in each zone, which are modeled by one single Gaussian and GMMs with 2, and 4 components for comparison purposes. During the online phase, the WD-GSF is applied on the test RSSI vector 100runs to (i) Smooth the RSSI vector, and; (ii) Model the RSSI vector at each step with one single Gaussian and a GM with 2 components. Fig. 2(a) illustrates the smoothed RSSI resulting from WD-GSF and KF-based algorithm in Reference [16]. Two scenarios are applied on the data based on the BD between Gaussian distribution of the current test data (resulting from WD-GSF and KF model) and Gaussian distribution of all 64 zones. The BD between different combination of trained models (with different number of Gaussian components within the GMMs) and test data are computed for both scenarios. Table 1 and 2 show the accuracy of the two scenarios with different combination of n_c and m_c getting from the WD-GSF algorithm. The best achieved zone classification accuracy for first scenario and second scenario $(K = 5)$ are 62% and 95%, respectively. Based on these experiments, it is concluded that using 2 components within GMMs representing fingerprinting zones and 2 components of GMM provided by the WD-GSF (representing the probabilistic model of the real-time RSSI values) coupled with utilization of the BD and Scenario S2 provides the better result (95%) compared to modelling the real-time RSSI values with KF-based localization algorithm in Reference [16] (45% and 90%). Fig. 2(b) compares the Mean Square Error (MSE) in estimating the location resulting of two mentioned algorithm based on 100 runs.

4. CONCLUSION

In this paper, to deal with the fact that RSSI-based solutions are prone to multipath fading and drastic fluctuations in the indoor environment, the combinations of different number of components of Gaussian Mixture Model of fingerprinting and test data are used for indoor location determination by using the GSF with WD-based GMR and BD algorithm. The experimental results show that the performance of this method, in terms of accuracy is better than that of the common algorithm based on single Gaussian Model, KF based smoothing and Euclidean distance.

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